MATH 170 - Test 3 - Key

1) $f(x) = x^4 - 4x^3$

a) $f'(x) = 4x^3 - 12x^2$ exists everywhere.

Set $f'(x) = 0$

$f'(x) = 4x^2(x-3)$

$4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0$ OR $x = 3$

$f(0) = 0$ $\quad f(3) = 81 - 108 = -27$

So two critical pts: $(0,0)$ and $(3,-27)$

b)

interval	f' on interval	incr/decr
$(-\infty, 0)$	neg	decr
$(0,3)$	neg	decr
$(3,\infty)$	pos	incr

c) $(3,-27)$ is a local min

$(0,0)$ is neither

2) Show $\cos x = 3x - 5$ has at most one solution.

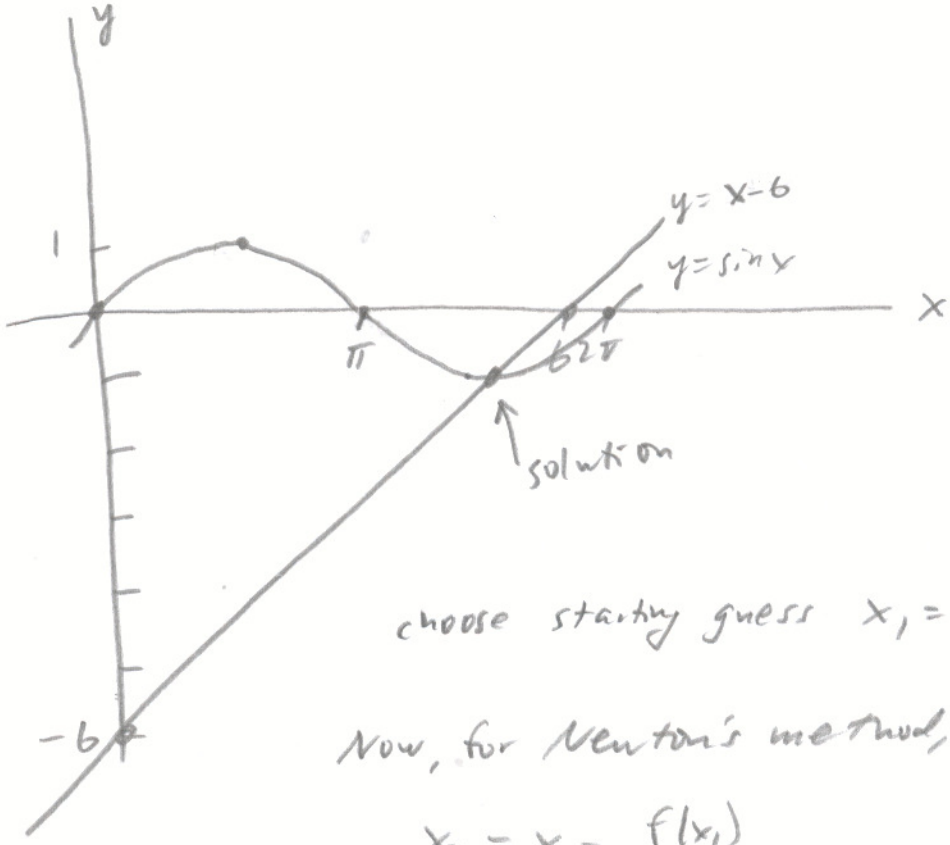
Let $f(x) = 3x - 5 - \cos x$. Show $f(x) = 0$ has at most one solution.

pf Assume otherwise, so assume $f(x) = 0$ has at least two distinct solutions - call them p and q. So $f(p) = f(q)$. So Rolle's Thm says $\exists$ a number c b/w p and q s.t. $f'(c) = 0$.

But $f'(x) = 3 + \sin x$, and $2 \le f'(x) \le 4$. So $f'(x) > 0$.

This contradicts $f'(c) = 0$. So $f(x) = 0$ has at most one solution. $\boxtimes$

3)



choose starting guess $x_1 = 5$

Now, for Newton's method,

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(x) = \sin x - x + 6$

$f'(x) = \cos x - 1$

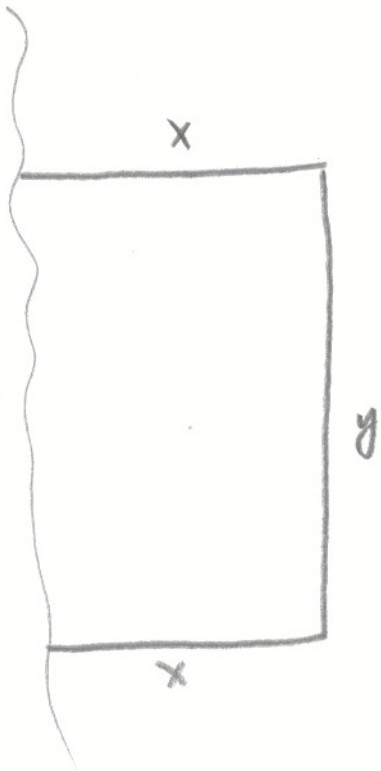
$$x_2 = 5 - \frac{\sin 5 - 5 + 6}{\cos 5 - 1}$$

$$x_2 = 5 - \frac{\sin 5 + 1}{\cos 5 - 1}$$

4) MVT: $f'(c) = \dfrac{f(b) - f(a)}{b-a} = \dfrac{120 \text{ mi}}{90 \text{ min}} = \dfrac{4}{3} \dfrac{\text{mi}}{\text{min}} \cdot \dfrac{60 \text{ min}}{\text{hr}} = 80 \dfrac{\text{mi}}{\text{hr}}$

so on the journey, there was a time at which my velocity is $80 \frac{\text{mi}}{\text{hr}}$. Since the speed limit is $75 \frac{\text{mi}}{\text{hr}}$ or less, I was speeding.

5)



$2x+y=800 \longrightarrow y=800-2x$

Maximize $A=xy$

$A(x)=x(800-2x)$

$A(x)=800x-2x^2$

domain is $[0,400]$

$A(0)=0 \qquad A(400)=320000-2(160000)=0$

Now, $A'(x)=800-4x$ exists everywhere.

Set $A'(x)=0$. $800-4x=0$

$x=200 \Rightarrow y=400$

$A(200)=800\cdot 200-2(200)^2$

$=160000-2\cdot 40000$

$=80{,}000$

We enclose $80{,}000\ m^2$ of land.

Two lengths of 200 ft each

One length of 400 ft.

6) $f(x) = \frac{2}{\sqrt[3]{x}} + 3\sin 4x - 6\cos 3x + \frac{8}{x} - 5e^{-9x}$

$f(x) = 2x^{-1/3} + 3\sin 4x - 6\cos 3x + \frac{8}{x} - 5e^{-9x}$

$F(x) = 2 \cdot \frac{3}{2} x^{2/3} + 3\left(\frac{-1}{4}\cos 4x\right) - 6\left(\frac{1}{3}\sin 3x\right) + 8\ln|x| - 5 \cdot \left(\frac{-1}{9}e^{-9x}\right) + C$

$F(x) = 3x^{2/3} - \frac{3}{4}\cos 4x - 2\sin 3x + 8\ln|x| + \frac{5}{9}e^{-9x} + C$

7) $f(x) = 2x - 4$, interval $[2, 6]$

$\Delta x_i = \frac{b-a}{n} = \frac{6-2}{n} = \frac{4}{n}$

$x_i^* = a + i\Delta x$

$= 2 + i \cdot \frac{4}{n} = 2 + \frac{4i}{n}$

$f(x_i^*) = 2\left(2 + \frac{4i}{n}\right) - 4$

$= \frac{8i}{n}$

$\text{area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$

$\text{area} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i}{n} \cdot \frac{4}{n}$

$= \lim_{n\to\infty} \frac{32}{n^2} \sum_{i=1}^{n} i$

$= \lim_{n\to\infty} \frac{32}{n^2} \cdot \frac{n(n+1)}{2}$

$= \lim_{n\to\infty} \frac{32n^2}{2n^2}$ (using dominant terms)

$= \frac{32}{2} = 16$