MATH 170 - Test 2 - Key - Version 4

1) a) Prove: $\frac{d}{dx}(\tan x) = \sec^2 x$

pf $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

b) Prove $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

pf Let $y = \arctan x$. We want to prove $\frac{dy}{dx} = \frac{1}{1+x^2}$

Since $y = \arctan x$, we have $\tan y = x$. Now diff both sides wrt x: $\sec^2 y \frac{dy}{dx} = 1$. Now, $\cos^2 y + \sin^2 y = 1$,

so $1 + \tan^2 y = \sec^2 y$

$1 + x^2 = \sec^2 y$

so $(1+x^2)\frac{dy}{dx} = 1$

so $\frac{dy}{dx} = \frac{1}{1+x^2}$

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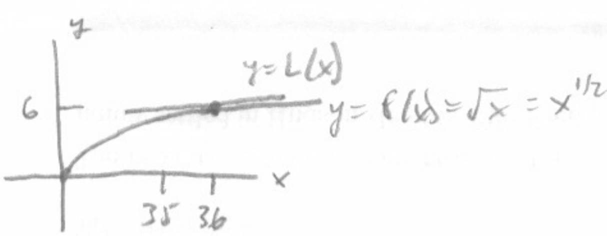
2)



$y = L(x) = mx + b$

$m = f'(36)$. Now $f'(x) = \frac{1}{2}x^{-1/2}$

so $m = f'(36) = \frac{1}{2}\cdot(36)^{-1/2} = \frac{1}{2}\cdot\frac{1}{6} = \frac{1}{12}$

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{1}{12}(x - 36)$

$y - 6 = \frac{1}{12}x - 3$

$y = \frac{1}{12}x + 3 = L(x)$

Now $\sqrt{35} = f(35) \approx L(35)$

$= \frac{1}{12}\cdot 35 + 3$

$= \frac{1}{12}(36-1) + 3$

$= 3 - \frac{1}{12} + 3$

$= 6 - \frac{1}{12} = 5\frac{11}{12}$

so $\sqrt{35} \approx 5\frac{11}{12} = \frac{71}{12}$

This is an overestimate since the tangent line lies above the curve.

3) $f(x) = e^{\cos^2 5x}$

$f'(x) = e^{\cos^2 5x} \cdot \frac{d}{dx}(\cos^2 5x)$

$= e^{\cos^2 5x} \cdot 2\cos 5x \ \frac{d}{dx}(\cos 5x)$

$= e^{\cos^2 5x} \cdot 2\cos 5x \cdot (-\sin 5x) \cdot \frac{d}{dx}(5x)$

$= e^{\cos^2 5x} \cdot 2\cos 5x \cdot (-\sin 5x) \cdot 5$

$= -10\, e^{\cos^2 5x} \cdot \cos 5x \cdot \sin 5x$

4) $y(t) = -16t^2 + 32t + 128$ y in feet, t in seconds.

a) max height when $y'(t) = 0$

$y'(t) = -32t + 32$

set $y'(t) = 0 \Rightarrow t = 1$

now, $y(1) = -16 + 32 + 128$

$= 16 + 128$

$= 144$

so the max height is 144 ft above the ground

b) speed when $y = 0$?

$0 = y(t) = -16t^2 + 32t + 128$

$= -16(t^2 - 2t - 8)$

$= -16(t-4)(t+2)$

so $t - 4 = 0$ OR $t + 2 = 0$

$t = 4$ or $t = -2$

nonsense

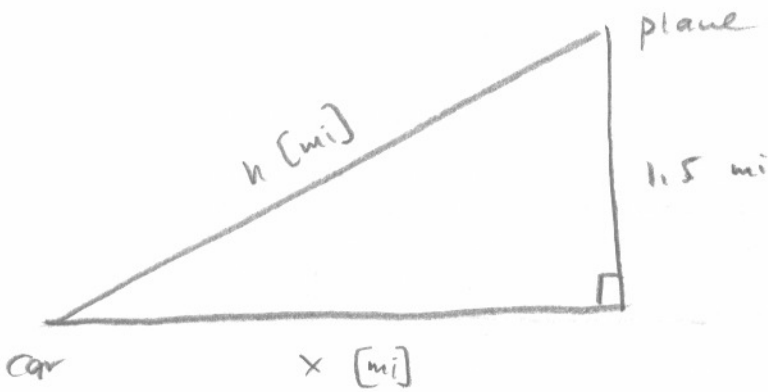
so $t = 4$.

now, $v(4) = y'(4) = -32 \cdot 4 + 32$

$= 32(-3)$

$= -96$

so the speed of the ball when it hits the ground is $96 \frac{ft}{sec}$.



t = time [hr]

Find $\frac{dx}{dt}$ when $h = 2.5$ and $\frac{dh}{dt} = -200$

$x^2 + (1.5)^2 = h^2$ Diff both sides wrt t:

$$2x \frac{dx}{dt} + 0 = 2h \frac{dh}{dt}$$

$$x \frac{dx}{dt} = h \frac{dh}{dt}$$ when $h = 2.5$, $x = 2$

$$2 \frac{dx}{dt} = 2.5(-200)$$

$$\frac{dx}{dt} = \frac{2.5}{2}(-200)$$

$$= \frac{5}{4}(-200)$$

$$= -250$$

This is the rate at which the horizontal distance between the plane and car is changing

Since the plane is approaching the car at a horizontal speed of 320 mph, we conclude the car is travelling at $320 - 250 = 70$ mph.