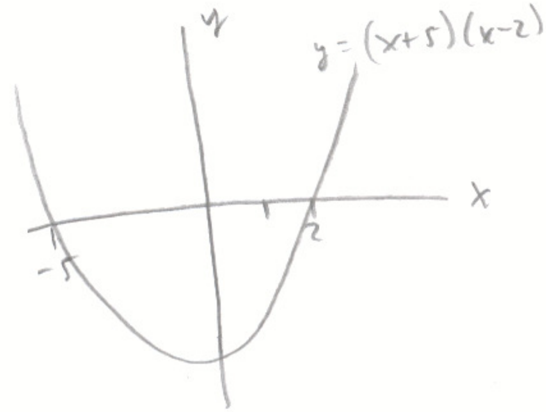
MATH 170 - Test I - Version B

1) $f(x) = (x^2+3x-10)^{-1/2} = \dfrac{1}{\sqrt{x^2+3x-10}}$



Need $x^2+3x-10>0$

$(x+5)(x-2)>0$

so need $x<-5$ OR $x>2$

So domain is $(-\infty,-5) \cup (2,\infty)$

2) $f(x) = \cos^5 3x$

Let $g(x) = 3x$

Let $h(x) = \cos x$

Let $p(x) = x^5$

So $f(x) = p(h(g(x)))$.

3) $3+3\cos x - 2\sin^2 x = 0$

$3+3\cos x - 2(1-\cos^2 x) = 0$

$3+3\cos x - 2 + 2\cos^2 x = 0$

$2\cos^2 x + 3\cos x + 1 = 0$

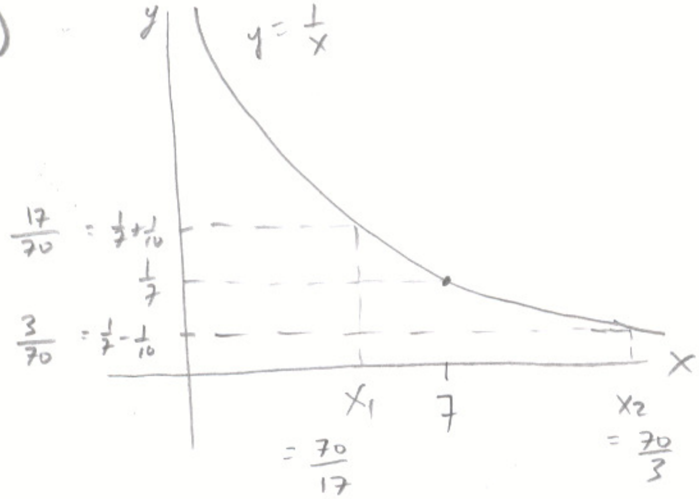
$(2\cos x + 1)(\cos x + 1) = 0$

$2\cos x + 1 = 0$ OR $\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$ OR $\cos x = -1$

$x = \pm\frac{2\pi}{3} + 2k\pi$ OR $x = \pi + 2k\pi$, where k is an integer.

4)

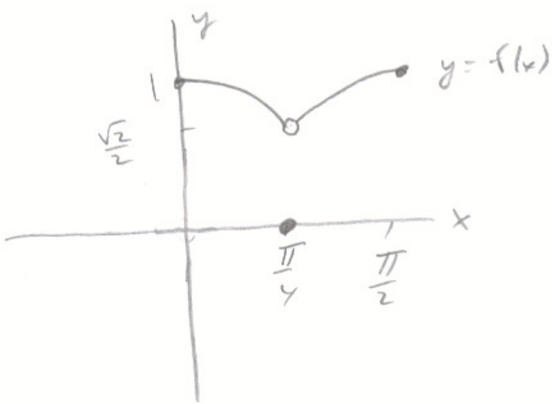


$$\begin{array}{r} 17 \\ 7 \\ \hline 119 \end{array}$$

$$\delta_1 = 7 - \frac{70}{17} = \frac{119-70}{17} = \frac{49}{17} \qquad \delta_2 = x_2 - 7 = \frac{70}{3} - 7 = \frac{70}{3} - \frac{21}{3} = \frac{49}{3}$$

Since $\frac{49}{17} < \frac{49}{3}$, we choose $\delta = \frac{49}{17}$

5) Let $f(x) = \begin{cases} \cos x & \text{if } x < \frac{\pi}{4} \\ 0 & \text{if } x = \frac{\pi}{4} \\ \sin x & \text{if } x > \frac{\pi}{4} \end{cases}$



$\lim_{x \to \frac{\pi}{4}^-} f(x) = \frac{\sqrt{2}}{2}$ and $\lim_{x \to \frac{\pi}{4}^+} f(x) = \frac{\sqrt{2}}{2}$. These one-sided limits are equal.

Thus $\lim_{x \to \frac{\pi}{4}} f(x)$ exists and equals $\frac{\sqrt{2}}{2}$

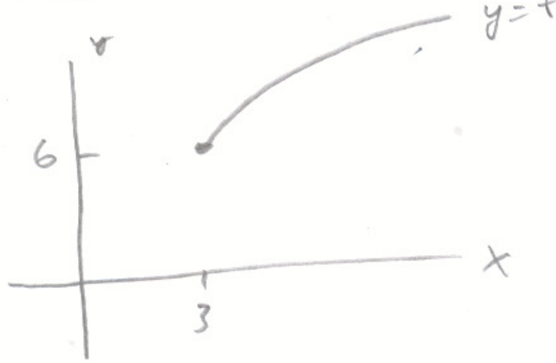
6) $f(x) = x^2 + 5$

Then $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^2 + 5] - [x^2 + 5]}{h}$

$= \lim_{h \to 0} \frac{(x^2 + 2hx + h^2 + 5) - (x^2 + 5)}{h}$

$= \lim_{h \to 0} \frac{2hx + h^2}{h} = \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} (2x+h) = 2x$

7) $f(x) = 6 + \sqrt{x-3}$



range is $[6, \infty)$

To find $f^{-1}(x)$:

1) $y = f(x) = 6 + \sqrt{x-3}$

2) interchange x's and y's: $x = 6 + \sqrt{y-3}$

3) Solve for y: $x - 6 = \sqrt{y-3}$

$(x-6)^2 = y - 3$

$y = (x-6)^2 + 3$

So $f^{-1}(x) = (x-6)^2 + 3$ w/ domain $[6, \infty)$.