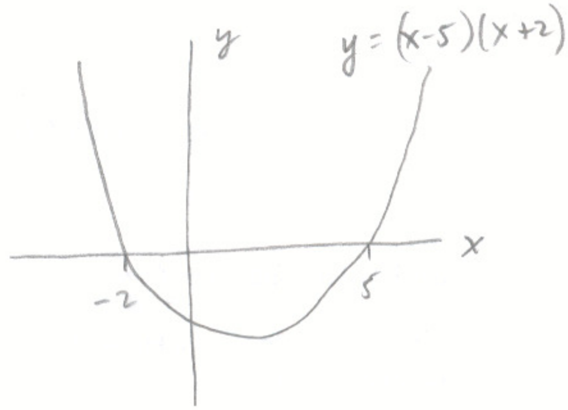
MATH 170 - Test I - version A

1) $f(x) = (x^2-3x-10)^{-1/2} = \dfrac{1}{\sqrt{x^2-3x-10}}$



need $x^2-3x-10 > 0$

$(x-5)(x+2) > 0$

so need $x > 5$ OR $x < -2$

So domain is: $(-\infty, -2) \cup (5, \infty)$

2) $f(x) = \cos^3 5x$

Let $g(x) = 5x$

Let $h(x) = \cos x$

Let $p(x) = x^3$

So $f(x) = p(h(g(x)))$

3) $3 + 3\sin x - 2\cos^2 x = 0$

$3 + 3\sin x - 2(1-\sin^2 x) = 0$

$3 + 3\sin x - 2 + 2\sin^2 x = 0$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

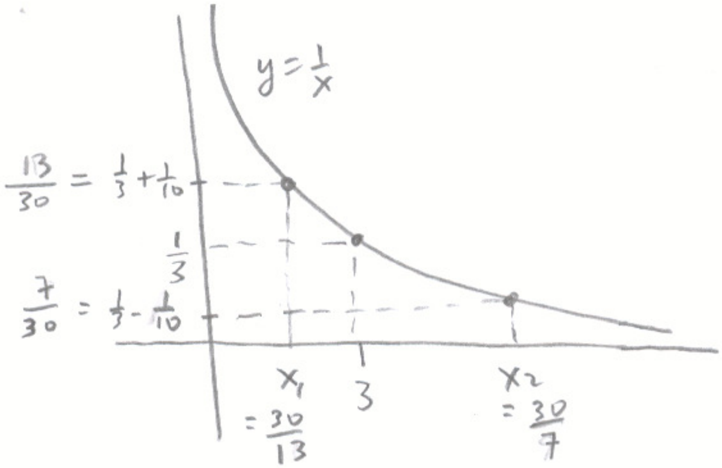
$2\sin x + 1 = 0$ OR $\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$ OR $\sin x = -1$

$$\left.\begin{array}{l} x = -\frac{\pi}{6} + 2k\pi \\ \quad\text{OR} \\ x = \frac{7\pi}{6} + 2k\pi \end{array}\right\} \text{ OR } \; x = \frac{3\pi}{2} + 2k\pi, \text{ where } k \text{ is an integer.}$$

So $x = -\frac{\pi}{6} + 2k\pi$ OR $x = \frac{7\pi}{6} + 2k\pi$ OR $x = \frac{3\pi}{2} + 2k\pi$, where k is an integer

4)



$\delta_1 = 3 - x_1 = \frac{39}{13} - \frac{30}{13}$

$\delta_1 = \frac{9}{13}$

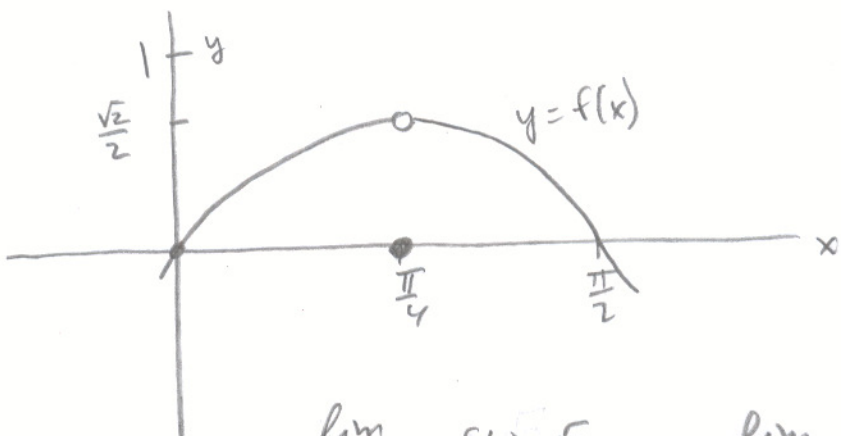
$\delta_2 = x_2 - 3$

$= \frac{30-21}{7}$

$\delta_2 = \frac{9}{7}$

Since $\frac{9}{13} < \frac{9}{7}$, we choose $\delta = \frac{9}{13}$

5) $f(x) = \begin{cases} \sin x & \text{if } x < \frac{\pi}{4} \\ 0 & \text{if } x = \frac{\pi}{4} \\ \cos x & \text{if } x > \frac{\pi}{4} \end{cases}$



$\lim_{x \to \frac{\pi}{4}^-} f(x) = \frac{\sqrt{2}}{2}$ and $\lim_{x \to \frac{\pi}{4}^+} f(x) = \frac{\sqrt{2}}{2}$

The one-sided limits are equal. Thus $\lim_{x \to \frac{\pi}{4}} f(x)$ exists and equals $\frac{\sqrt{2}}{2}$.

6) $f(x) = x^2+3$

Then $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^2+3]-[x^2+3]}{h}$

$= \lim_{h\to 0} \frac{x^2+2hx+h^2+3-x^2-3}{h}$

$= \lim_{h\to 0} \frac{2hx+h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} (2x+h) = 2x.$

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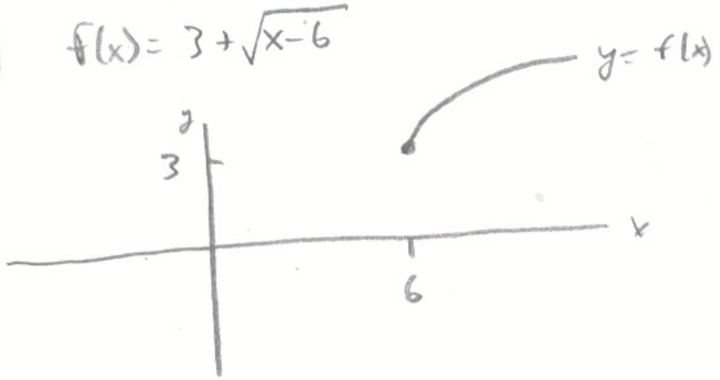
7) $f(x) = 3+\sqrt{x-6}$



range is $[3, \infty)$

To find $f^{-1}(x)$

1) $y = f(x) = 3+\sqrt{x-6}$

2) interchange x's and y's: $x = 3+\sqrt{y-6}$

3) solve for y: $\sqrt{y-6} = x-3$

$$y-6 = (x-3)^2$$

$$y = (x-3)^2+6$$

So $f^{-1}(x) = (x-3)^2+6$ w/ domain $[3, \infty)$.