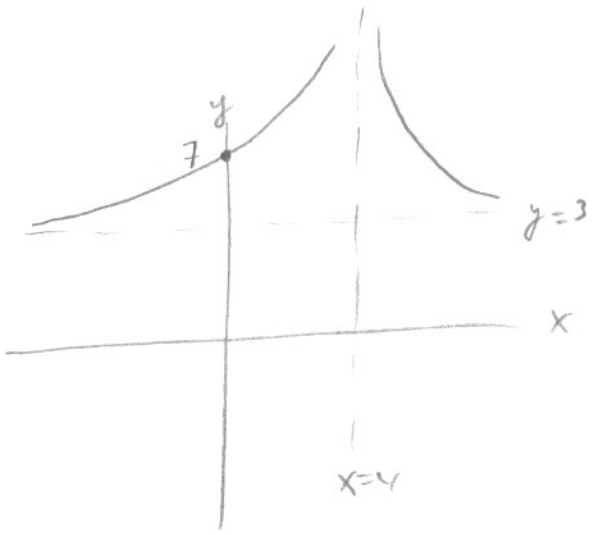
MATH 170 - Quiz 4 - Key

1) $\lim_{x\to\infty} f(x) = 3$

$\lim_{x\to-\infty} f(x) = 3$

$f(0) = 7$

$\lim_{x\to 4} f(x) = \infty$



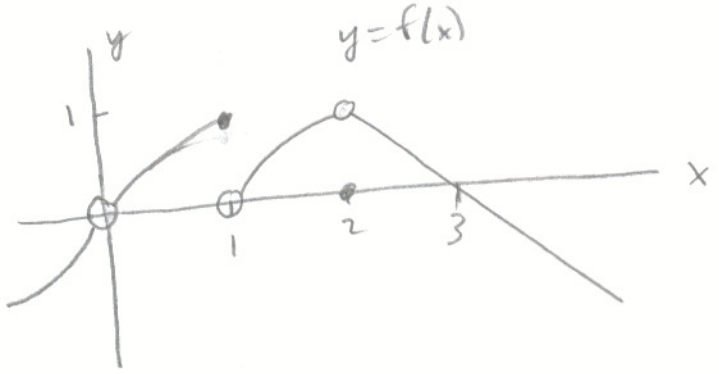
$f(x) = \dfrac{3x^2 + b}{(x-4)^2}$ satisfies all conditions except $f(0) = 7$

$7 = f(0) = \dfrac{b}{16}$ so $b = 16 \cdot 7 = 112$

So $f(x) = \dfrac{3x^2 + 112}{(x-4)^2}$

2)

$$f(x) = \begin{cases} \sin x & \text{if } x < 0 \\ \sin x & \text{if } 0 < x \le 1 \\ \log_2 x & \text{if } 1 < x < 2 \\ 0 & \text{if } x = 2 \\ 3 - x & \text{if } x > 2 \end{cases}$$



a) f is not continuous at $x=0$ b/c $f(0)$ DNE

b) f is not continuous at $x=1$ b/c $\lim_{x\to 1^-} f(x) = \sin 1 \neq 0 = \lim_{x\to 1^+} f(x)$

Thus $\lim_{x\to 1} f(x)$ DNE. So f cannot be continuous at $x=1$.

c) f is not continuous at $x=2$ b/c $\lim_{x\to 2} f(x) = 1 \neq 0 = f(2)$

d) f is continuous at $x=3$ b/c $\lim_{x\to 3} f(x) = 0 = f(3)$

3) Show $x = \cos x$ has a solution.

pf Let $f(x) = x - \cos x$. So showing $x = \cos x$ has a solution is equivalent to showing that $f(x) = 0$ has a solution.

Now, $f(0) = 0 - \cos 0 = 0 - 1 = -1$

and $f(\pi) = \pi - \cos\pi = \pi - (-1) = \pi + 1 > 0$.

Since 0 is b/w $f(0)$ and $f(\pi)$ and $f(x)$ is continuous everywhere, the IVT tells us that $f(c) = 0$, where c is some number in the interval $(0, \pi)$.

Thus $c - \cos c = 0$

and thus $c = \cos c$.

Thus the equation $x = \cos x$ has a solution, namely c, where $c \in (0, \pi)$. ☒

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4) $y = f(x) = 3x - x^2$ at $x = -1 = x_1$

$$\text{slope} = m = \lim_{x\to -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim_{x\to -1} \frac{(3x - x^2) - (-3-1)}{x+1} = \lim_{x\to -1} \frac{-x^2 + 3x + 4}{x+1}$$

$$= \lim_{x\to -1} \frac{(x+1)(-x+4)}{x+1} = \lim_{x\to -1} (-x+4) = 1 + 4 = 5.$$ Note $f(-1) = -3 - 1 = -4 = y_1$

So tangent line is $y - y_1 = m(x - x_1)$

$$y + 4 = 5(x+1)$$