MATH 170 – Section 009 – Quiz 2

You may work with other class members on this quiz, but you may not receive assistance from people not in MATH 170 (Section 009). You must show all of your work to receive full credit. Do all your work on other sheets of paper and be sure to staple all the pieces of paper together or YOU WILL GET A ‘ZERO’ ON THE QUIZ. Do not use decimal approximations unless asked to do so. Your work on this quiz must be handed in by Thursday, 8 September 2005 at 1900. GOOD LUCK!

1) Let \( b > 0 \) with \( b \neq 1 \). Given the fact that the equations

\[
\log_b x = y
\]

and

\[
b^y = x
\]

are equivalent, prove the following properties of logarithms:

a) \( \log_b (b^x) = x \) for every real number \( x \)

b) \( \log_b x = x \) for all positive numbers \( x \)

c) \( \log_b (xy) = \log_b x + \log_b y \) for all positive numbers \( x \) and \( y \)

d) \( \log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y \) for all positive numbers \( x \) and \( y \)

e) \( \log_b (x^r) = r \log_b x \) for all positive numbers \( x \) and all real numbers \( r \)

2) Find the inverse of the function

\[
f (x) = \sqrt{2 - x}.
\]

Then plot the graphs of \( y = f (x) \) and \( y = f^{-1} (x) \) on the same set of axes.

3) Use the technique of Section 2.1 of the textbook to estimate the slope of the line tangent to \( y = \sqrt{x} \) at \( x = 9 \). You should compute at least four different slopes. Then use Maple to graph both \( y = \sqrt{x} \) and its tangent line at \( x = 9 \) on the same set of axes.
4) Let

\[ f(x) = \begin{cases} 
1 - x & \text{if } 0 \leq x < 1 \\
2 & \text{if } x = 1 \\
x - 1 & \text{if } 1 < x \leq 2 \\
4 - x & \text{if } x > 2.
\end{cases} \]

Tell whether or not the following limits exist. If a limit exists, give its value. Explain your answers fully.

a) \( \lim_{x\to1} f(x) \)

b) \( \lim_{x\to2} f(x) \)

5) Evaluate:

a) \( \lim_{x\to\pi^+} \cot x \)

b) \( \lim_{x\to\pi^-} \cot x \)

Do these limits exist? Explain.