

Abstract

We are concerned with the numerical solution of partial differential equations (PDEs) in two spatial dimensions discretized by Hermite collocation. In order to efficiently solve the resulting systems of linear algebraic equations, we elect to use the Bi-CGSTAB method of van der Vorst (1992). As in all conjugate gradient methods, the selection of a suitable preconditioning matrix is necessary for the rapid convergence we seek. Our choice of preconditioner is based upon a Red-Black numbering scheme which makes the method amenable to parallel processing. In addition, we investigate shifting the collocation points away from the so-called “Gauss points” in an effort to further accelerate convergence.

For a number of symmetric model problems, we are able to derive analytic formulae for the eigenvalues that control the rate at which the Bi-CGSTAB method converges. These formulae, which depend upon the location of the collocation points, can be utilized to determine where the collocation points should be placed in order to make the Bi-CGSTAB method converge as quickly as possible.

We test our method on the PDEs which govern multiphase subsurface flow and contaminant transport. We find that our method, even implemented serially and at the “Gauss points”, is almost always significantly faster than GMRES/ILU, a popular method for the solution of such problems.

We also implement our method on the Cray T3E, a supercomputer that can take full advantage of the parallelism inherent in the algorithm. We find that for sufficiently large problems, we obtain almost linear speedup. In addition, we study the scalability of the algorithm via its isoefficiency function.