Geometry, Topology, and Complexity of Virtual Knots

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June 24-27, 2012

AAAS Pacific Division Conference
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Quick Review

Given any knot, one can extract the associated 2-complex, by setting labels for each arc, changing to a new label when the arc stops due to the knot undercrossing itself.
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The spherical diagram, a tiling of the sphere with the squares of the 2-complex of the knot, is reduced if one can do no moves to reduce the tiling further, or, no square is touching its inverse— the fold move— or is vertex adjacent to its own square— allowing a diamond move— in turn allowing fold moves.
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That a given knot’s 2-complex is (DR) implies that the knot’s 2-complex is aspherical.
The question of whether a 2-complex is aspherical or not is recursively undecidable\textsuperscript{[3]}.

However, reduced spherical diagrams of certain LOIs have been found, and one can study these LOIs in an attempt to understand their construction.
Are All Long Knots DR?

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If the knot is prime, meaning that the knot is not the sum of any other knots (and thus does not contain any knots within it) and alternating, then the associated 2-complex is DR.
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If the associated LOI of the knot is injective, boundary reduced, and has no subloi, then the knot is DR.
Construction for non-DR Knots

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[Diagram of non-DR Knots]

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Are All Long Virtual Knots DR?

No.
No.
The smallest\textsuperscript{[3]} example:
No.  
The smallest\(^3\) example:
Are all long alternating virtual knots DR?

This question is an open problem.
Harlander, Jens. “REU Lectures on Combinatorial Topology.”


Acknowledgments

Thank you Boise State University REU 2012 in Mathematics, Boise State University, National Science Foundation (DMS 1062857), and Louisiana State University!

Thank you Dr. Oliver Dasbach, Dr. Jens Harlander, Dr. Liljana Babinkostova, Dr. Marion Scheepers, and my teammates!