

Finding topologically interesting points in βX under the continuum hypothesis

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Let X be a completely regular topological space. A compactification of X is a compact space Y together with a homeomorphic embedding $c : X \rightarrow Y$ such that $\overline{c[X]} = Y$. The Stone-Ćech compactification βX of X has the following property: whenever $f : X \rightarrow [0, 1]$ is a continuous function from X to the closed unit interval, f continuously extends to a function $\hat{f} : \beta X \rightarrow [0, 1]$. βX contains a homeomorphic copy of X , plus many new points that are added in the construction of βX . The collection of these new points is called the remainder, or growth, and is denoted $\beta X \setminus X$.

Beginning in the 1950's, topologists were trying to understand the structure of the remainder. For example, they asked: is $\beta X \setminus X$ homogeneous? (That is, given two points $p, q \in \beta X \setminus X$, is there a homeomorphism taking p to q ?) The complex structure of βX for even quite simple spaces X meant that answers to such questions often depended on the continuum hypothesis (CH). W. Rudin proved in 1956 that, assuming CH, $\beta X \setminus X$ is not homogeneous for a certain class of spaces X . In 1967, Z. Frolík was able to prove Rudin's result without invoking CH. However, Frolík's proof that $\beta X \setminus X$ is not homogeneous was "noneffective" in the sense that it did not produce any topologically interesting points.

Remote points are one example of such topologically interesting points. A remote point of X is a point $p \in \beta X \setminus X$ such that p is not in the closure (in βX) of any nowhere-dense subset of X . van Douwen in 1981 gave an effective proof, using remote points, of the non-homogeneity of $\beta X \setminus X$ (for another class of topological spaces X). It is an open question whether CH implies that every non-pseudocompact space with the countable chain condition (ccc) has remote points. We have shown that under CH, a ccc, nonpseudocompact product of ccc spaces each of weight ω_2 has remote points. In this talk we outline our proof, which uses elementary submodels and an analysis of ccc Boolean algebras to impose a hierarchy on the basic open sets in a space.

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