

STATISTICAL PROPERTIES OF THE REGULARIZED LEAST SQUARES FUNCTIONAL AND A HYBRID LSQR NEWTON METHOD FOR FINDING THE REGULARIZATION PARAMETER: APPLICATION IN IMAGE DEBLURRING AND SIGNAL RESTORATION

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Abstract

Image deblurring or signal restoration can be formulated as a data fitting least squares problem, but the problem is found to be ill-posed and regularization is needed, hence introducing the need to find a regularization parameter. We study the properties of the regularized least squares functional

$$\|Ax - b\|_{W_b}^2 + \|D(x - x_0)\|_{W_x}^2$$

for the solution of discretely ill-posed systems of equations. It was recently shown to follow a χ^2 distribution when the *a priori* information x_0 on the solution is assumed to represent the mean of the solution x . Of course for image deblurring no prior information is available, but possibly information on mean value of the right hand side b is available, which yields again a χ^2 distribution, but one that is non-central. These results can be used to design a Newton method, using a hybrid LSQR approach, for the determination of the optimal regularization parameter λ when the weight matrix is $W_x = \lambda^2 I$. Numerical results using test problems demonstrate the efficiency of the method, particularly for the hybrid LSQR implementation. Results are compared to another statistical method, the unbiased predictive risk (UPRE) algorithm. We also illustrate the results for image deblurring and a real data seismic signal deblurring problem.

