

This test has page 1 - 5. Take a moment to make sure you've got them all.

No Calculators Allowed; No Reference Materials; Just You and Your Pencil and Eraser.

- 1 State the formal definition of $\lim_{n \rightarrow \infty} a_n = L$, where $-\infty < L < \infty$.
- 2 State the famous *Monotone Sequence Theorem*.
- 3 Let $a_n = \frac{5n - 8}{3n + 11}$. Find $\lim_{n \rightarrow \infty} a_n$, and then show the scratch work for the *N*-recipe.

- 4 Find the area of the region bounded above and below by the curves $18x - y = -12$ and $y = 12x^2$ for and lying between the vertical lines $x = 0$ and $x = 3$.

- 5 Let \mathcal{R} be the region enclosed by the graphs of $y = x^2$, $y = 9$ and $x = 1$ (note that \mathcal{R} does not touch the y -axis). Imagine the solid \mathcal{S} generated by revolving the region \mathcal{R} about the line $x = 3$. Set up the integrals \mathbf{X} and \mathbf{Y} , where \mathbf{X} is a “ dx ” integral and \mathbf{Y} is a “ dy ” integral, both giving the *volume* of the solid \mathcal{S} . *Do not evaluate your integrals.* Make sure the grader can easily make out which one’s \mathbf{X} and which one’s \mathbf{Y} .

6 For each of the following expressions, find its value, if it has a value (that is, if it converges). If it does not have a value, say so, and explain briefly.

(a) $\lim_{n \rightarrow \infty} (n^3 + 1)^{-1}$

(b) $\lim_{n \rightarrow \infty} (3^n)^{-1}$

(c) $\sum_{n=0}^{\infty} (3^n)^{-1}$

(d) $\sum_{n=2}^{\infty} (3^n)^{-1}$

7 For each of the following expressions, explain whether it converges.

(a) $\sum_{n=1}^{\infty} (3n)^{-1}$

(b) $\lim_{n \rightarrow \infty} \left(-\frac{2}{3}\right)^{-n}$

(c) $\sum_{n=1}^{\infty} (3n^2)^{-1}$

- 8 Show steps in finding the limit of the sequence $\mathbf{a}_n = \left(1 + \frac{1}{3n}\right)^n$.